

# A Model of Secular Stagnation: A Quantitative Analysis for an Endowment Economy with Fiscal Policy<sup>1</sup>

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## \* 1. Introduction

Since the Global Financial Crisis, the United States and Europe have been reliving the Japanese experience of sluggish productivity growth, low inflation, and low to negative natural rates of interest. This phenomenon, in the literature, has been coined as “secular stagnation”, first proposed by Hansen (1939) and reintroduced by Summers (2014). The secular stagnation hypothesis was revived due to the consequences of the GFC, since the aftermath of the GFC followed similar patterns ensuing the Great Depression. This idea is formalized herein using an overlapping generation model first developed by Samuelson (1958) and applied in the context of secular stagnation by Eggertsson, Mehrotra, and Robbins (2019) among others. As will be shown, contrary to NK models, the natural rate of interest in the model presented is determined by fundamental secular forces affecting the supply and demand for loans, which need not be reverted after a shock has dissipated. This leads to chronically binding ZLB, which has substantial implications for economic policy. In the standard ZLB literature, the ZLB episode is reverted as the exog-

enous shock that caused it phases out. Conversely, in the framework presented, even if the exogenous shock no longer holds significance, the economy can, under certain circumstances, find itself in a stable steady state in which the equilibrium real interest rate consistent with output at potential is permanently negative. Fiscal policy can, if aggressive enough, move an economy out of the secular stagnation steady state into a steady state characterized by a positive natural rate of interest.

Since the analysis presented herein can be characterized as an analysis of the supply and demand of loans, any policy that leads to a redistribution or a reduction in the oversupply of savings and an increase in the natural rate of interest can move an economy away from the secular stagnation steady state. On the other hand, a policy that causes the opposite – an increase in savings – would lead to an exacerbation of the symptoms of secular stagnation. Contrary to the representative agent framework of the standard NK model, any force altering the relative supply of savings and investments leads in the model analysed to a change in the natural rate of interest. We show that a slowdown in population growth, an increase in income inequal-

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ity, and a tightening of collateral constraints all have negative effects on the natural rate of interest.

The main contributions of this paper are a) proposing an analytical framework that lays out a theoretical background to the determination of natural rates of interest in line with (Eggertsson, Mehrotra, and Robbins 2019), b) presenting a quantitative analysis based on the theoretical model and c) simulating fiscal and macroprudential policy actions in countering the forces driving an economy into a secular stagnation steady state. The paper is organized in the following manner. Section 2 highlights our motivation and relates the paper to existing literature. Section 3 lays out the fundamental framework of an overlapping generations model in an endowment economy with no production and no nominal frictions. Section 4 incorporates fiscal policy into the baseline model. Section 5 presents simulations on how changes in exogenous variables, namely changes in population growth, the collateral constraint, taxes, and income inequality affect endogenous variables of the model, particularly consumption patterns of different generations and the natural rate of interest. Section 6 concludes the paper.

## 2. Motivation and Related Literature

Since the GFC, secular stagnation has become a pressing issue in the global economy and especially in the euro area. The theoretical and quantitative background provided in this article can aid in understanding the consequences of the overhang of savings over investments (specifically savings in the northern states of the euro area and investments in the southern periphery), a phenomenon that Baldwin and Giavazzi (2015), and Rodriguez-Palenzuela and Dees (2016) have documented in detail. The purpose of this article is to present a deeper and structured understanding of the underlying causes of the savings-investment imbalance. A future application of the issue discussed within the context of the EU could give a clearer picture of how the problem is affecting specific countries and

how it may differ from other regions of the world. Furthermore, discussing the potential solutions to the overhang of savings over investments provides practical information on how to mitigate the negative effects of secular stagnation. Highlighting the policies and strategies that the theoretical model suggests allows us to comprehend the potential solutions to this economic problem and how these solutions can later be adapted in the context of a monetary union, in which member states operate with different fiscal policies. Overall, this article can help us understand the current economic challenges and potential solutions to them. It can also serve as an opportunity to discuss the role of the EU in addressing this issue and the impact of the EU's policies on the member states economies.

It might come as a surprise that in the decade following the GFC there has not been any shift in economic policy towards models allowing for a secular stagnation steady state. Representative agent general equilibrium models by design do not allow the possibility of a permanently negative natural rate of interest, which unfortunately has less to do with economic reality than with the fundamental aspects and limitations of those models. Based on those models, the natural rate of interest is estimated as the interest rate that prevails in a representative agent economy without nominal frictions. Using this methodology, Neiss and Nelson (2003) estimate the real interest rate and the real interest rate gap as an inflation indicator. Smets and Wouters (2003) develop a similar model, which focuses on the euro area rather than the United Kingdom. Using the same framework to analyse policy options at the ZLB, Christiano, Eichenbaum, and Rebelo (2011) focus on fiscal policy multipliers, and Eggertsson and Woodford (2003) emphasize forward guidance in helping an economy out of a liquidity trap if the current policy is able to manage a change in the expectations about the future conduct of policy. In a resembling fashion, even though Eggertsson and Krugman (2012) incorporate financial frictions and analyse the debt deleveraging aspects of the financial crisis, which

→ also affects the natural rates of interest in the model discussed in this paper, the representative agent framework they base their analysis on inversely relates the interest rate directly to the discount factor of households, eliminating the possibility of a permanently negative real natural rate of interest.

Recent literature has sought to address these gaps by reevaluating the concept of the natural rate of interest and its connection to secular stagnation. Baldi and Harms (2017) discuss the empirical and literary evidence supporting the relationship between the natural rate of interest and secular stagnation, emphasizing structural factors like demographic shifts and productivity slowdowns contributing to low interest rates. Their analysis aligns with the broader hypothesis of secular stagnation being a persistent economic condition. Papetti's (2021) research highlights demographic changes as a pivotal factor affecting the natural rate of interest. Papetti's work suggests that long-term demographic trends, often overlooked in representative agent models, are crucial for understanding shifts in the natural rate. This analysis resonates with current economic trends, where aging populations across many advanced economies pose significant challenges to monetary policy frameworks and interest rate settings. This perspective is complemented by Bonam et al. (2018), who examine the natural rate of interest from monetary and financial viewpoints, highlighting the significant influence of monetary policy and financial cycles. Taylor (2017) challenges the conventional interpretation further by proposing alternative macroeconomic models that incorporate income distribution and debt dynamics, offering a distinct narrative on the low-growth environment. Moreover, Di Bucchianico (2023) revisits the theory of the negative natural rate of interest within the frameworks of liquidity trap and secular stagnation theories, emphasizing the relevance of this concept.

Our analysis thus necessitates a shift away from representative agent models, as many authors had previously suggested. Bystrov's (2020) work for

example challenges the foundational assumptions of representative agent models by arguing that the natural and unnatural rates of interest, in practice, may not be as distinct as previously thought. This assertion raises critical questions about the models' capacity to accurately predict interest rate behaviours, especially when contrasted with real-world observations where such distinctions often blur under the influence of policy interventions and market dynamics. Similarly, Ponomarenko's (2018) investigation reveals how credit cycles significantly impact the estimation of the natural rate, highlighting a potential oversight in traditional models that may abstract away the financial market's complexities. This perspective is particularly relevant when considering the empirical evidence of post-2008 financial crisis interest rate behaviours. These studies illuminate the complexity of estimating the natural rate of interest and the limitations of representative agent models in capturing the full spectrum of factors that influence real-world interest rate behaviours.

The model presented herein is a model of natural rates of interest and its determinants based on the determinants of the demand and supply of loans. It is thus in its fundamental aspect related closely to the seminal work by Wicksell (1936). Backhouse and Boianovsky (2016) draw connections to Wicksell's seminal work, providing a historical perspective that underscores the enduring significance of the negative natural rate of interest in understanding today's economic challenges. Considering that the analysis presented herein is based on analysing the determinants of the demand and supply of loans, our paper is related to the literature on the "global savings glut". The concept of the "global savings glut" as proposed by Bernanke (2005) offers a compelling explanation for the persistently low levels of long-term real interest rates worldwide. He argues that "a significant increase in the global supply of saving helps to explain ... the relatively low level of long-term real interest rates in the world today". This framework has been further elaborated upon by Mian,

Straub, and Sufi (2020a), who connect the savings glut with income inequality and the dissaving behaviours of non-rich individuals and governments, particularly in the context of the United States. They argue that “instead of financing investment, this saving glut has been associated with dissaving by the non-rich and dissaving by the government”. A similar analysis can be applied to the euro area in line with Baldwin and Giavazzi (2015), emphasizing the widespread applicability of this phe-

ing, and paying back debt. On the other hand, the decrease in investments is mainly related to technological advancements, which essentially leads to a reduction in the demand for physical capital and a fall in the prices of capital goods. This perspective is crucial in understanding the evolving dynamics of global savings and investment patterns.

Furthermore, Mian, Straub, and Sufi (2020b) develop a very similar model to the one presented

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nomenon. Recently, the discourse around the global savings glut has evolved, with newer research shedding light on demographic developments, investment platforms, and the interplay between savings, investment, and infrastructure financing. For instance, a thesis by Pettersen and Håland (2016) delves into the demographic underpinnings of the savings glut, particularly focusing on China and the US. Their work provides a numerical simulation that highlights the significant impact of aging populations on savings behaviours, offering a nuanced understanding of the factors contributing to the global savings glut. Moreover, Arezki et al. (2016) explore the transition from a global savings glut to financing infrastructure through the advent of investment platforms. This IMF Working Paper illustrates how the excess global savings can be channelled into productive investment, particularly in infrastructure, which has the potential to mitigate some of the adverse effects of the savings glut on global economic growth. Summers (2016) attributes the increase in savings to longer life expectancy, more resistance to debt, more uncertainty, household deleverag-

ing, and paying back debt. On the other hand, the decrease in investments is mainly related to technological advancements, which essentially leads to a reduction in the demand for physical capital and a fall in the prices of capital goods. This perspective is crucial in understanding the evolving dynamics of global savings and investment patterns.

Furthermore, Mian, Straub, and Sufi (2020b) develop a very similar model to the one presented in Eggertsson, Mehrotra, and Robbins (2019), in which they focus primarily on income inequality and financial liberalization as an explanation of a phenomenon they label “indebted demand”. This phenomenon is characterized by low or negative natural rates of interest and high levels of private debt. Wang and Luo (2023) extend the dialogue to consider how varying degrees of financial liberalization impact the broader economic stability, thus affecting the underlying mechanisms of indebted demand. Their analysis suggests that the relationship between financial liberalization and inequality is complex, influenced significantly by income distribution patterns. Simultaneously, the study by Kim, Hsieh, and Lin (2021) explores the nuanced interplay between financial liberalization and income inequality, emphasizing the mediating role of political institutions. This research indicates that the outcomes of financial liberalization on income inequality are highly contingent on the nature of political frameworks, introducing an additional layer of complexity to understanding economic disparities and their implications for indebted demand.



The model discussed in this paper is also in line with the measurements of the natural rates of interest by Laubach and Williams (2003) and Holston, Laubach, and Williams (2017), where they explicitly state that “explanations for this decline [of the natural rates of interest] include shifts in demographics, a slowdown in trend productivity growth, and global factors affecting real interest rates”, all of which can be incorporated into different extensions of our baseline model. Lansing’s (2016) forward-looking analysis also acknowledges the persistent decline in the natural rate, attributing it to structural factors like aging populations and productivity slowdowns. In a similar fashion, Manrique and Marques (2004) suggest that real-world economic conditions—ranging from fiscal policies to global economic shocks—play a significant role in determining the natural rate, raising questions about the adequacy of models that do not account for these factors. Wynne and Zhang (2018) follow in the steps of Laubach and Williams (2003) and estimate the time-varying natural rate jointly with the output gap and trend potential output growth for the world as a whole. They find that the world natural rate of interest has indeed been trending down for the past few decades. Similarly, Rachel and Smith (2015) estimate that the long-term real interest rates across the world have fallen by about 450 basis points over the past 30 years.

### 3. The Model

We start by developing a simple endowment economy to illustrate the determinants of natural rates of interest in a three-generational OLG model. Households live for three periods. They are born in period one (young), they enter middle age in period two (middle age) and retire in period three (old). In this simplified model we assume no physical capital. The only way of an inter-temporal transfer of wealth is by borrowing and lending, although there is a limit to borrowing that is incorporated as an exogenous time-varying constant as in Eggertsson and Krugman (2012). Moreover, indi-

viduals only receive income in periods two and three. Therefore, the young must borrow to be able to consume and the middle-aged must save to be able to smooth their consumption throughout their lifetime. In period three the individuals consume their savings and any income they receive. The old do not save, they just consume what is left. This means that we assume the availability of a risk-free interest-bearing asset that allows the transfer of wealth. Since a household’s objective is to maximize its lifetime utility, we consider a logarithmic utility function that can be written as:

$$\max_{c_t^y, c_{t+1}^m, c_{t+2}^o} \mathbb{E}_t [\log(C_t^y) + \beta \log(C_{t+1}^m) + \beta^2 \log(C_{t+2}^o)] \quad (1)$$

where  $C_t^y$  denotes consumption of the household when it is young,  $C_{t+1}^m$  denotes consumption when it is middle-aged, and  $C_{t+2}^o$  is the consumption of the household when it is old. The reason for choosing a logarithmic function is that it is a continuous, concave and twice differentiable function so it is easy to work with. Moreover, the most accepted measures of the coefficient of relative risk aversion lie between 1 and 3.

Borrowing takes place through a risk-free asset that lasts for one period only. Since income is received only in the second and third period of a household’s lifetime, when young, the entire consumption is financed by borrowing. This means that the young household issues a bond to cover its entire consumption. In the second period pays back the amount borrowed plus accrued interest on that first period bond (that has reached maturity), issues a new bond (or saves) and receives income. In the third period the old household pays the amount borrowed in the second period (or consumes the savings) plus interest and receives income. We assume that the household cannot die indebted. Lastly, as stated, we consider an exogenous time-varying borrowing limit. Having this in mind, we can formulate the budget constraints that the households born at time  $t$  face at each stage of their lifetime in the following manner:

$$C_t^y = B_t^y \tag{2}$$

$$C_{t+1}^m = Y_{t+1}^m - (1 + r_t)B_t^y + B_{t+1}^m \tag{3}$$

$$C_{t+2}^o = Y_{t+2}^o - (1 + r_{t+1})B_{t+1}^m \tag{4}$$

$$(1 + r_t)B_t^i \leq D_t \tag{5}$$

$B_t^i$ , where  $i = y, m$ , is the bond issued by a household in one of the first two periods of its lifetime at an interest rate  $r_t$  and  $r_{t+1}$ . Equation (2) illustrates the budget constraint of the young, equation (3) shows the budget constraint of the middle-aged household that, as stated, receives income  $Y_{t+1}^m$ , pays back the amount borrowed when young plus interest  $(1 + r_t)B_t^y$  and borrows  $B_{t+1}^m$ , or equivalently saves  $-B_{t+1}^m$  for the future. Equation (4) illustrates that consumption of the old household is constrained by the endowment received in the last period and the repayment of the debt, or the consumption of savings from the previous period  $+(1 + r_{t+1})B_{t+1}^m$ . The inequality in equation (5) reiterates that the borrowing in each period is constrained by an exogenous constant  $D_t$  as in Eggertsson and Krugman (2012). Let us assume that for the young generation the constraint is binding<sup>2</sup>. Mathematically expressed, that:

$$C_t^y = B_t^y = \frac{D_t}{1 + r_t} \tag{6}$$

As can be seen from equation (6), consumption of the young at time  $t$  is dependent on the ability of the household to repay the debt when they reach period two (with respect to the current interest rate faced by the household at which they issue the bond). Thus, a decrease in the market interest rate results in a decrease in future costs and an increase in consumption. Furthermore, an increase in the exogenous debt constraint  $D_t$  results in an increase in young age consumption and vice versa. The old at any given time  $t$  consume all their in-

come, which can be simply expressed as:

$$C_t^o = Y_t^o - (1 + r_{t-1})B_{t-1}^m \tag{7}$$

since their consumption is based on the market interest rates and bond prices they faced in the previous period (when they were in their middle age). The middle-aged at time  $t$  however must decide on how much to consume now and how much to consume in the next period when they are old. Their consumption pattern is therefore determined by the standard Euler equation.

The middle-aged now base their decisions only on the current and next period consumption (since their young age consumption has already been determined by the borrowing constraint in the previous period). Imagine that middle-aged household decreases  $C_t^m$  by an infinitesimal amount  $\Delta C$  and then uses the additional saving and income to raise  $C_{t+1}^o$  by  $(1 + r_t)\Delta C$ . This change does not affect the present value of the individual's lifetime consumption stream. If the individual is optimizing, the utility cost and benefit of the change must be equal. The marginal contributions of  $C_t^m$  and  $C_{t+1}^o$  to lifetime utility are  $\frac{1}{C_t^m}$  and  $\beta \frac{1}{C_{t+1}^o}$ , respectively. As we let  $\Delta C$  approach 0, the utility cost of the change approaches  $\frac{1}{C_t^m}\Delta C$  and the utility benefit approaches  $\beta \frac{1}{C_{t+1}^o}(1 + r_t)\Delta C$ . As mentioned, if the individual is optimizing, the costs and benefits must be equalized. Mathematically expressed, that:

$$\frac{1}{C_t^m}\Delta C = \beta \frac{1}{\mathbb{E}_t C_{t+1}^o}(1 + r_t)\Delta C \tag{8}$$

Cancelling the  $\Delta C$ 's results in the standard Euler equation, such that:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1 + r_t}{C_{t+1}^o} \tag{9}$$

For household  $i$  we can formulate the optimization problem in a similar fashion, such that we maximize: →

<sup>2</sup> For the constraint to be binding, it must be the case that  $D_t = \frac{Y_{t+1} + \frac{Y_{t+2}}{1 + r_{t+1}}}{1 + \beta(1 + \beta)}$ .

$$\max_{C_t(i), C_{t+1}(i), C_{t+2}(i)} \mathbb{E}_t [\log(C_t(i)) + \beta \log(C_{t+1}(i)) + \beta^2 \log(C_{t+2}(i))] \quad (10)$$

subject to:

$$C_t(i) = B_t(i) \quad (11)$$

$$C_{t+1}(i) = Y_{t+1}(i) - (1 + r_t)B_t(i) + B_{t+1}(i) \quad (12)$$

$$C_{t+2}(i) = Y_{t+2}(i) - (1 + r_{t+1})B_{t+1}(i) \quad (13)$$

$$(1 + r_t)B_t(i) \leq D_t \quad (14)$$

We first formulate the intertemporal budget constraint by combining equations (11)–(13). We do this by expressing  $B_{t+1}(i)$  from equation (13) and substituting into equation (12). The same is done for  $B_t(i)$  in equation (12) and substituted into equation (11). We thus achieve the intertemporal budget constraint (without  $i$ ) as:

$$\mathcal{L} = U[\log(C_t) + \beta \log(C_{t+1}) + \beta^2 \log(C_{t+2})] + \lambda_t \{-C_t - [(C_{t+1} - Y_{t+1})(1 + r_t)^{-1}] - [(C_{t+2} - Y_{t+2})(1 + r_{t+1})^{-1}(1 + r_t)^{-1}]\} \quad (16)$$

Taking first order conditions with respect to consumption in each period we end up with:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t} = 0 &\Leftrightarrow \lambda = \frac{1}{C_t} \\ \frac{\partial \mathcal{L}}{\partial C_{t+1}} = 0 &\Leftrightarrow \lambda = \beta \frac{(1 + r_t)}{C_{t+1}} \\ \frac{\partial \mathcal{L}}{\partial C_{t+2}} = 0 &\Leftrightarrow \lambda = \beta^2 \frac{(1 + r_{t+1})(1 + r_t)}{C_{t+2}} \end{aligned} \quad (17)$$

This means that:

$$\frac{1}{C_t} = \beta \frac{(1 + r_t)}{C_{t+1}} = \beta^2 \frac{(1 + r_{t+1})(1 + r_t)}{C_{t+2}} \quad (18)$$

Also, we can rearrange to show that:

$$\begin{aligned} C_{t+1} &= \beta(1 + r_t)C_t \\ C_{t+2} &= \beta(1 + r_{t+1})C_{t+1} \\ C_{t+2} &= \beta^2(1 + r_t)(1 + r_{t+1})C_t \end{aligned} \quad (19)$$

Earlier in equation (14) we assumed that the exogenous constraint is binding. For this to be true,

we can show that by substituting the results from equation (19) into the budget constraint in equation (16) and substituting for  $C_t$  the expression  $\frac{D_t}{1 + r_t}$ , so that:

$$\begin{aligned} C_t + \frac{\{\beta(1 + r_t)C_t - Y_{t+1}\}}{1 + r_t} + \frac{\{\beta^2(1 + r_t)(1 + r_{t+1})C_t - Y_{t+2}\}}{(1 + r_t)(1 + r_{t+1})} &= 0 \\ C_t + \beta C_t - \frac{Y_{t+1}}{1 + r_t} + \beta^2 C_t - \frac{Y_{t+2}}{(1 + r_t)(1 + r_{t+1})} &= 0 \\ C_t(1 + \beta + \beta^2) &= \frac{Y_{t+1}}{1 + r_t} + \frac{Y_{t+2}}{(1 + r_t)(1 + r_{t+1})} \\ D_t(1 + \beta + \beta^2) &= Y_{t+1} + \frac{Y_{t+2}}{1 + r_{t+1}} \end{aligned} \quad (20)$$

For the constraint to be binding, it must be the case that:

$$D_t = \frac{Y_{t+1} + \frac{Y_{t+2}}{1 + r_{t+1}}}{1 + \beta(1 + \beta)} \quad (21)$$

Let us now assume that the size of each generation is given exogenously by  $N_t$ . The growth rate of each new generation is given by  $1 + g_t = \frac{N_t}{N_{t-1}}$ , where  $N_{t-1}$  is the size of the currently middle-aged generation ( $N_{t+1}$  would signify the size of the young in the next period). For there to be equilibrium in the bond market, we have to assume that the savings of the middle-aged generation equals the borrowing of the young. This means that  $N_t B_t^y = -N_{t-1} B_t^m$ . Or, given the growth rate, that:

$$(1 + g_t)B_t^y = -B_t^m \quad (22)$$

Considering we have all the ingredients necessary to analyse the determinants of the natural rate of interest, we can proceed to derive a straightforward endogenous solution to the interest rate from the model. This is in direct contrast to the standard Euler equation causation in which we try to assess and determine lifetime consumption patterns based on movements in the interest rate and the discount factor. Equation (22) represents the equilibrium in the loans and savings market. The left-

hand side is the demand for loans (formed by the young generation at time  $t$ ), and the right-hand side is the supply of loans (the middle-aged supply the loans again at time  $t$ ). We denote demand as  $L_t^d$  and supply as  $L_t^s$ . Assuming that the constraint for the young households is binding, we can formulate the loan demand by substituting  $\frac{D_t}{1+r_t}$  for  $B_t^y$  in the left-hand side of equation (22) as:

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t \tag{23}$$

To derive loan supply, we need to utilize equations (3) and (4) and substitute them into equation (9). Moreover, since we solve this equation from the perspective of the middle-aged at time  $t$ , it is necessary to reformulate the budget constraints in equations (3) and (4) by moving them a period back in the following manner:

$$C_t^m = Y_t^m - (1 + r_{t-1})B_{t-1}^y + B_t^m \tag{24}$$

$$C_{t+1}^o = Y_{t+1}^o - (1 + r_t)B_t^m \tag{25}$$

Substituting equations (24) and (25) into the middle-aged Euler equation (9) and substituting for  $B_{t-1}^y$  the expression  $\frac{D_{t-1}}{1+r_{t-1}}$ , we get that:

$$\begin{aligned} \beta(1 + r_t)[Y_t^m - D_{t-1} + B_t^m] &= Y_{t+1}^o - (1 + r_t)B_t^m \\ \beta(1 + r_t)[Y_t^m - D_{t-1}] - Y_{t+1}^o &= -\beta(1 + r_t)B_t^m - (1 + r_t)B_t^m \\ \beta Y_t^m - \beta D_{t-1} - \frac{Y_{t+1}^o}{1 + r_t} &= -\beta B_t^m - B_t^m \\ \beta(Y_t^m - D_{t-1}) - \frac{Y_{t+1}^o}{1 + r_t} &= -(\beta + 1)B_t^m \\ \beta(Y_t^m - D_{t-1}) - \frac{Y_{t+1}^o}{1 + r_t} &= -(\beta + 1)B_t^m \\ -B_t^m &= \frac{\beta}{1 + \beta}(Y_t^m - D_{t-1}) - \frac{1}{1 + \beta} \frac{Y_{t+1}^o}{1 + r_t} \end{aligned} \tag{26}$$

This equation corresponds to the right-hand side of equation (22), which, as discussed earlier, represents the supply of loans in this endowment economy. We can express the equilibrium interest

rate by equating loan supply and demand as follows:

$$\begin{aligned} \frac{(1 + g_t)}{1 + r_t} D_t &= \frac{\beta}{1 + \beta}(Y_t^m - D_{t-1}) - \frac{1}{1 + \beta} \frac{Y_{t+1}^o}{1 + r_t} \\ \frac{(1 + g_t)}{1 + r_t} D_t + \frac{1}{1 + \beta} \frac{Y_{t+1}^o}{1 + r_t} &= \frac{\beta}{1 + \beta}(Y_t^m - D_{t-1}) \\ (1 + g_t)D_t + \frac{1}{1 + \beta} Y_{t+1}^o &= \frac{\beta}{1 + \beta}(Y_t^m - D_{t-1})(1 + r_t) \tag{27} \\ \frac{(1 + g_t)D_t}{Y_t^m - D_{t-1}} + \frac{1}{1 + \beta} \frac{Y_{t+1}^o}{Y_t^m - D_{t-1}} &= \frac{\beta}{1 + \beta}(1 + r_t) \\ 1 + r_t &= \frac{1 + \beta(1 + g_t)D_t}{\beta(Y_t^m - D_{t-1})} + \frac{1}{\beta} \frac{Y_{t+1}^o}{Y_t^m - D_{t-1}} \end{aligned}$$

From equation (27) it is evident that the natural rate of interest is determined by a host of factors. This includes the income over a household's lifetime, the discount factor, the debt constraint, and the population growth rate. Moreover, changes in the relative incomes of the middle and old-aged can also affect the interest rate. An increase in population growth and an increase in the borrowing limit tend to have positive effects on the natural rate of interest. The dynamics of an increase in population growth are rather intuitive – as the number of young increases relative to the middle-aged, the demand for loans rises [see equation (23)]. For markets to clear, the natural rate of interest must increase as well. A relaxation of the borrowing limit, which leads to an increase in the demand for loans, has similar effects on the natural rates of interest, although the transition dynamics differ. A decrease in the incomes of the middle-aged relative to those of the old similarly leads to an increase in the natural rate of interest, since a decrease in the relative middle-aged income translates into a decrease in savings, which in turn puts upward pressure on the interest rate.

Conversely, a decrease in the rate of population growth, the debt constraint and an increase in inequality should result in the opposite effects – a reduction in the natural rate of interest.

With regard to the borrowing limit, a contraction leads to a decrease in the demand for loans,



→ which in turn necessitates a reduction in the natural rate of interest to maintain equilibrium in the loans and savings market. This dynamic is also in line with the model developed by Eggertsson and Krugman (2012). However, in the representative agent framework, once the borrowing limit returns to its initial value (after the deleveraging shock abates), the natural rate of interest increases and equals the inverse of the discount factor.

In the OLG framework the dynamics differ since households are both borrowers and savers at different stages of their lifetimes. A reduction in the borrowing of young households at time  $t$  implies that in the next period, when the household becomes a saver, each household will have higher disposable income and consequently higher savings [see equation (26)]. This leads to an increase in loan supply until it equalizes loan demand at the lower natural rate of interest (which can eventually be negative depending on the magnitude of borrowing limit contraction).

Notice that there is no downside limit on the values of the natural rate of interest. Nothing is stopping the natural rate of interest derived from being negative. Lastly, forces such as population growth need not be reverted in the foreseeable future and thus, we could expect the effects on natural rates of interest to be long-lasting. We now incorporate fiscal policy into the baseline model to show how fiscal and macroprudential policy can negate the negative effects of decreasing population growth and increasing income inequality.

#### 4. The Model with Fiscal Policy

Having monetary policy constrained at the ZLB, the literature suggests that economic policy decision-making should shift its attention towards fiscal policy. The framework developed in this paper allows us to analyse different instruments of fiscal such as taxes and government spending, and macroprudential policy. These instrument and policies allow the government and macroeconomic stability authorities to target the savings of mid-

dle-aged households and the collateral constraints of the young, indirectly affecting loans and savings markets, consumption patterns and consequentially the natural rates of interest. Fiscal policy can be simply incorporated into our baseline model with some elementary adjustments to the budget constraints of households and by newly defining a government budget constraint.

To see how fiscal policy can affect households' decision-making, let us revise the equations presented in section 3. We maximize the same utility function given in equation (10), but now subject to:

$$C_t^y + T_t^y = B_t^y \tag{28}$$

$$C_{t+1}^m + (1 + r_t)B_t^y = Y_{t+1}^m - T_{t+1}^m - B_{t+1}^m \tag{29}$$

$$C_{t+2}^o = Y_{t+2}^o + (1 + r_{t+1})B_{t+1}^m - T_{t+2}^m \tag{30}$$

Public debt now figures in the asset market clearing condition.

$$N_t B_t^y + N_{t-1} B_t^g = -N_{t-1} B_t^m \tag{31}$$

where  $B_t^m$  is government debt. This means that loan demand is now formed not just by the young generation, but also by the government. Thus, the savings of the middle-aged generation need to cover both – the demands for loans by the young and the government:

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t + B_t^g \tag{32}$$

The supply for loans can be derived in a manner equivalent to equation (26), such that:

$$\beta(1 + r_t)[Y_{t+1}^m - D_{t-1} + B_t^m - T_t^m] = Y_{t+1}^o - (1 + r_t)B_t^m - T_{t+1}^o$$

$$\beta Y_t^m - \beta D_{t-1} - \beta T_t^m - \frac{Y_{t+1}^o - T_{t+1}^o}{1 + r_t} = -\beta B_t^m - B_t^m$$

$$\beta(Y_t^m - D_{t-1} - T_t^m) - \frac{Y_{t+1}^o - T_{t+1}^o}{1 + r_t} = -(\beta + 1)B_t^m \tag{32}$$

$$\beta(Y_t^m - D_{t-1} - T_t^m) - \frac{Y_{t+1}^o - T_{t+1}^o}{1 + r_t} = -(\beta + 1)B_t^m$$

$$L^s = -B_t^m = \frac{\beta}{1 + \beta}(Y_t^m - D_{t-1} - T_t^m) - \frac{1}{1 + \beta} \frac{Y_{t+1}^o - T_{t+1}^o}{1 + r_t}$$

If the government increases taxes for the middle-aged, the disposable income of middle-aged household decreases, their savings decrease and consequentially so does the loan supply. On the other hand, if the government imposes higher taxes on the old, aged households, savings of the middle-aged must increase to smooth out consumption over the household's lifetime. Lastly, we define a government budget constraint to determine asset market equilibrium:

$$T_t^m + B_t^g + \frac{1}{1+g_{t-1}}T^o + (1+g_t)Y_t^o = G_t + (1+r_{t-1})\frac{1}{1+g_{t-1}}B_{t-1}^g \quad (29)$$

Equating loan supply and demand similarly to equation (27) results in:

$$L^D = L^S$$

$$\frac{1+g_t}{1+r_t}D_t + B_t^g = \frac{\beta}{1+\beta}(Y_t^m - D_{t-1} - T_t^m) - \frac{1}{1+\beta}\frac{Y_{t+1}^o - T_{t+1}^o}{1+r_t}$$

$$\frac{1+g_t}{1+r_t}D_t + \frac{1}{1+\beta}\frac{Y_{t+1}^o - T_{t+1}^o}{1+r_t} + B_t^g = \frac{\beta}{1+\beta}(Y_t^m - D_{t-1} - T_t^m) \quad (35)$$

$$(1+g_t)D_t + \frac{1}{1+\beta}(Y_{t+1}^o - T_{t+1}^o) = \left[ \frac{\beta}{1+\beta}(Y_t^m - D_{t-1} - T_t^m) - B_t^g \right] (1+r_t)$$

$$(1+r_t) = \frac{(1+g_t)D_t}{\left[ \frac{\beta}{1+\beta}(Y_t^m - D_{t-1} - T_t^m) - B_t^g \right]} + \frac{1}{1+\beta} \frac{Y_{t+1}^o - T_{t+1}^o}{\left[ \frac{\beta}{1+\beta}(Y_t^m - D_{t-1} - T_t^m) - B_t^g \right]}$$

which means that the natural rate of interest is now determined by:

$$1+r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{(Y_t^m - D_{t-1} - T_t^m) - \frac{1+\beta}{\beta}B_t^g} + \frac{1}{\beta} \frac{Y_{t+1}^o - T_{t+1}^o}{(Y_t^m - D_{t-1} - T_t^m) - \frac{1+\beta}{\beta}B_t^g} \quad (31)$$

As can be seen from equation (32) and equation (36), an increase in public debt leads to an increase in the demand for loans and thus to an increase in the natural rate of interest. To have a permanent positive effect of public debt on the interest rate, the increase in the public debt must be permanent, otherwise the middle-aged would now increase savings (in anticipation of paying of the debt when old via taxation) and depress the interest rate back to its original level in the next period. Thus, with respect to fiscal policy, it is crucial to understand that it depends critically on households' expecta-

tions about future fiscal policy conduct. More specifically, it depends on the middle-aged households' expectations that it will not be taxed in the future to repay the public debt. Moreover, from the government constraint it is evident that the increase in public debt can be achieved by increasing public spending, transfers to the young or the middle-aged, or a combination of those. It is worth noting that it does not matter how the public debt is financed as long as it does not lead to a permanent increase in the relative supply and demand for loans (but rather the opposite). In section 4 we illustrate the effects that a change in the population growth, the collateral constraint, income inequality, public spending and taxation has on the natural rates of interest, consumption patterns, and public and private borrowing.

### 5. Simulations

We now turn to analysing the effects of exogenous variables and present a quantitative analysis based on the theoretical model presented in section 4. This is the main contribution of this paper, since Eggertsson, Mehrotra, and Robbins (2019) present the model described, but do not discuss a quantitative analysis based on that.

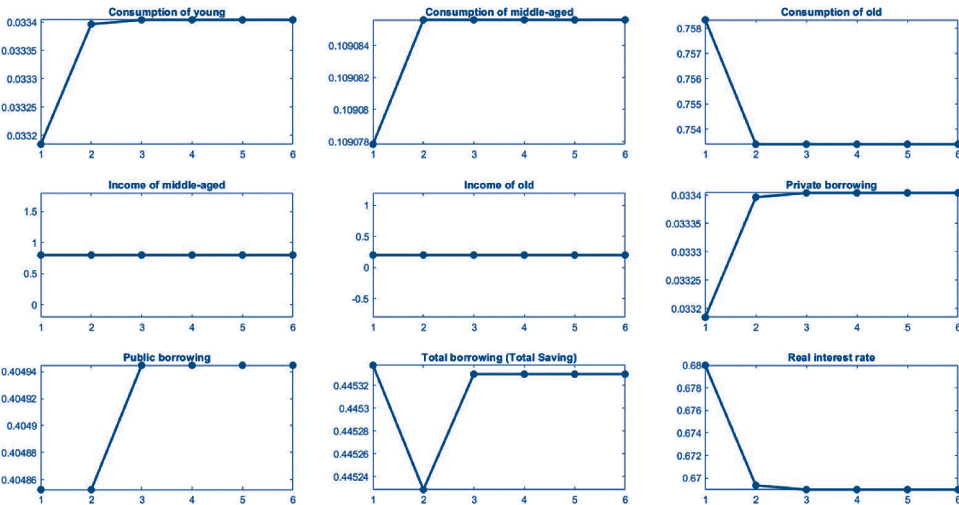
In testing the model, we assume that the young do not pay taxes and as previously described in the theoretical part, only the middle-aged and old receive income. Each generation lives for 20 years. The young generation is 15-35 years old, middle-aged is 35-55 and the old is 55-75. The steady state is computed in line with Eggertsson, Mehrotra, and Robbins (2019). Particularly, the initial steady state calibrates parameters and steady-state endogenous variables in such a way to match specific historic values of the year 1970. The final steady state assumes an exogenous change in one of the exogenous variables (e.g., population growth, income inequality, fiscal spending etc.) and solves endogenously for the rest of the model variables. The final steady state is solved using a loop process that tries different values of the interest rate until it

finds one that solves the system of equations. The initial steady-state target variables are average government spending, average government debt to output ratio, household debt to GDP ratio and the natural rate of interest. The exogenous variables are population growth rate and endowment of middle-aged cohort.

Average government spending is set to 0.2 (from FRED database). The average government and household debt values are 0.42 and 0.042, respectively, and can be found in Eggertsson, Mehrotra, and Robbins (2019) in table 4. The 20-year natural rate of interest is set to 0.68 (since the annual target is 0.0262, i.e., the 20-year rate is equal to  $1.0262^{20} - 1 = 0.68$ ). This value can be found in Eggertsson, Mehrotra, and Robbins (2019) table 5. With respect to the exogenous variables, the 20-year population growth is set at 0.22 (since the annual target is 0.01, over a 20-year period, this translates into a 22% population growth rate). Lastly, the endowment of the middle-aged is set arbitrarily to 0.8 (as a share of total income).

Below in figure 1 we can observe the transition dynamics of the model's endogenous variables with respect to a decline in the growth rate of population by 5%. As can be seen, in period 1, the decrease in the growth rate of population leads to a decrease in the demand for loans [see Equation (32)], since the young generation is the one that forms the demand for loans. At the same time, the decrease in loan demand is followed by a reduction in the natural rate of interest to clear asset markets. As the interest rate goes down, the consumption and borrowing of young go up [since we assume that the debt limit is binding from Equation (6) and does not change]. The consumption of the middle-aged also goes up [budget constraint Equation (29)]. This means that their savings must go down (because income is constant). Since the savings of the middle-aged go down and their consumption goes up, the consumption of the old must go down due to the fact that their disposable income decreases, and they cannot borrow (because we assumed that they exit the model debt-free).

**Figure 1** » A 5% decline in population growth rate



The effects of a 5% decline in population growth rate on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

**Table 1** » A 5% decline in population growth rate

Variable	Initial steady state	Final steady state
Consumption of young	0.0331846	0.0334038
Consumption of middle-aged	0.109078	0.109086
Consumption of old	0.758333	0.75341
Borrowing of young	0.0331846	0.0334038
Savings of middle-aged	0.445338	0.44533
Income of middle-aged	0.8	0.8
Income of old-aged	0.2	0.2
Natural rate of interest	0.68	0.668975
Government debt	0.404852	0.404945

The effects of a 5% decline in population growth rate on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

In the next period, the young enter middle age, the middle-aged enter old age, and the new young generation that enters the model is now smaller than the previous one. As can be seen, the decline in the interest rate leads to a relaxation of the borrowing limit of the current young generation [see Equation (21)]. Because the current interest rate at time  $t$  is dependent on the lagged value of the borrowing constraint  $D_{t-1}$  in Equation (36), in the next period the interest rate falls even lower. This leads to a slight increase in the borrowing of the new young generation and their consumption. Lastly, from the budget constraint of the government in Equation (34), with unchanged taxes and government spending, the borrowing of the government needs to increase to make up for the decline in population. This increases total savings, compensating for the decline in savings of the previous middle-aged generation.

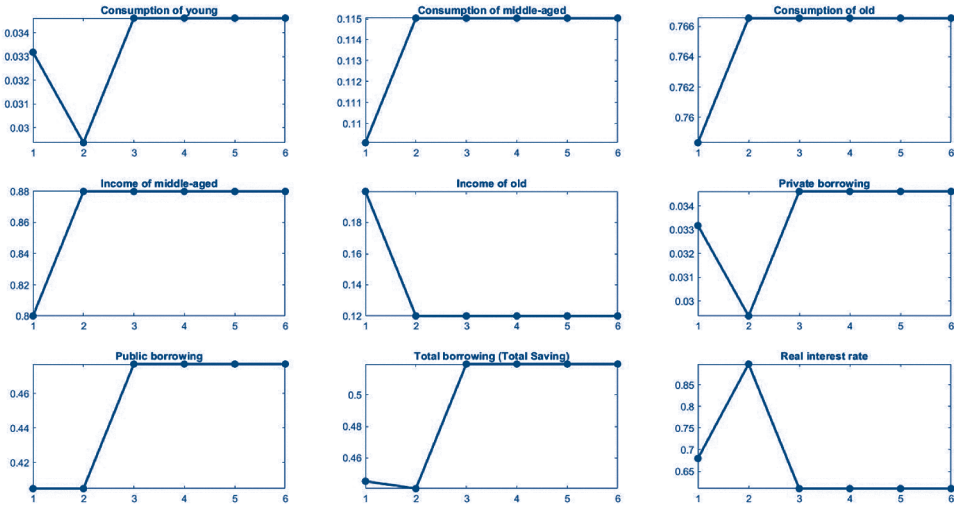
Table 1 illustrates the values of the endogenous variables in the initial and final steady state, respectively, and reiterates the results of the theoretical model. Most importantly, *it indicates the decline in the natural rate of interest caused by the decline in the growth rate of the population.*

Figure 2 breaks down the effects of a 10% in-

crease in the relative income between middle and old-aged cohorts (as can be seen in the second row of Figure 2). As the income of the middle-aged group increases, their consumption increases. As can be seen, even though there is a decline in the income of the old, there is an increase in their consumption. For this to occur, the middle-aged must reduce their lending to the young and redirect those assets as a saving for the old generation. This reduction in borrowing can only occur if the interest rate goes up (so that the young are not willing to borrow at that rate when they have no income and the middle-aged are enticed to save for old age). This then allows for a reduction in the consumption of the young and an increase in the consumption of the old. With private borrowing going down and public borrowing staying flat, total borrowing goes down. In the next period however, Equation (36) applies and the increase in middle-aged income is followed by an increase in loan supply.

This leads to a reduction in the interest rate (lower than at the initial state) and an increase in borrowing of the new young generation. Moreover, respecting the government's budget constraint, the reduction of the interest rate leads to an increase in

→ **Figure 2 » A 10% increase in relative income inequality**



The effects of a 10% increase in the relative incomes of middle-aged and old households on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

**Table 2 » A 10% increase in relative income inequality**

Variable	Initial steady state	Final steady state
Consumption of young	0.0331846	0.0346209
Consumption of middle-aged	0.109078	0.11503
Consumption of old	0.758333	0.766534
Borrowing of young	0.0331846	0.0346209
Savings of middle-aged	0.445338	0.519386
Income of middle-aged	0.8	0.88
Income of old-aged	0.2	0.12
Natural rate of interest	0.68	0.610302
Government debt	0.404852	0.477148

The effects of a 10% increase in the relative incomes of middle-aged and old households on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

public borrowing and combined with an increase of private borrowing, an increase in total borrowing. Table 2 below reaffirms our theoretical assumptions about the effects of increasing income inequality. **Increasing the income of the middle-**

**aged relative to the old households results in a decrease in the natural rate of interest.**

The following four figures and tables demonstrate macroprudential (Figure 3) and fiscal policy (Figures 4–6) effects on the model’s endogenous

variables. We consider macroprudential policy as a relaxation of the borrowing constraint of young households (for example in times of economic distress). To see those effects, consider a 40% increase in  $D_t$ .

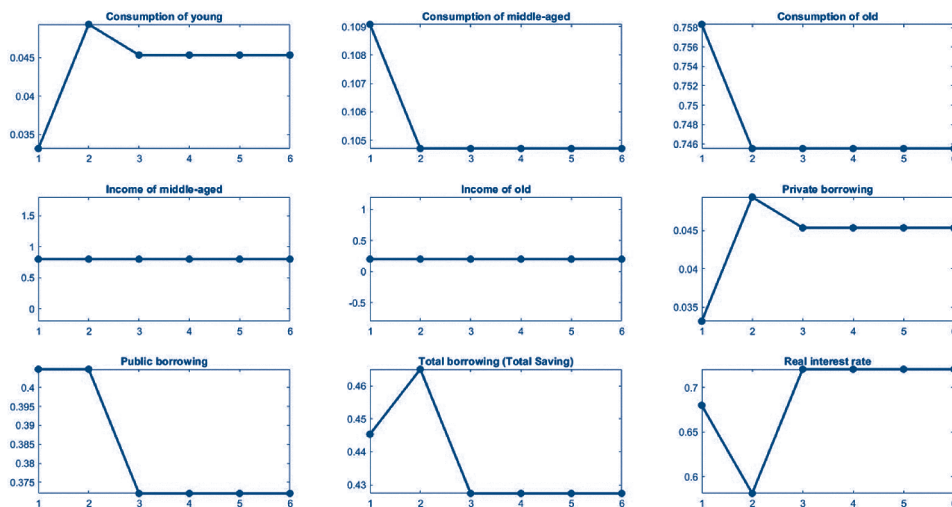
It is evident that a relaxation of the borrowing limit results in an instant increase in private borrowing. This increase coincides with an increase in the consumption of the young. The immediate effect of increased borrowing is a decrease in the middle-aged generation consumption.

Contrary to Figure 2, the consumption of the old in this case goes down too, which means that there has to be a reduction in savings by the middle-aged. For this to occur, the interest rate needs to temporarily go down (to demotivate middle-aged households from saving and motivate the young to borrow excessively). In the second period however, the interest rate starts increasing in line with Equation (36). This results in a reduction of private borrowing, a reduction of young household consumption and a reduction in public borrowing

according to Equation (34). Table 3 presents the results discussed. An increase of the borrowing limits leads to an increase in young household consumption but to a decrease in both, the consumption of the middle-aged and the old. Interestingly enough, the reduction in government debt outweighs the increase in young household spending. This indicates that total borrowing is lower in the final steady state, resulting in an interest rate increment that clears all markets. Table 3 thus reiterates our theoretical conclusions. **A relaxation of the borrowing constraint leads to an increase in the natural rate of interest.**

Figure 4 on the other hand demonstrates a 20% increase in government spending. According to Equation (34) we assume that an increase in government spending, *ceteris paribus*, results in a decrease in the natural rate of interest. Even though the natural rate of interest does end up lower in the final steady state, during its transition, the interest rate overshoots its steady-state level, and then returns to the final steady state. Because taxes do not

Figure 3 » A 40% increase in the borrowing constraint of the young



The effects of a 40% increase in the borrowing constraint of the young on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.



**Table 3 » A 40% increase in the borrowing constraint of the young**

Variable	Initial steady state	Final steady state
Consumption of young	0.0331846	0.0453638
Consumption of middle-aged	0.109078	0.10471
Consumption of old	0.758333	0.745534
Borrowing of young	0.0331846	0.0453638
Savings of middle-aged	0.445338	0.427405
Income of middle-aged	0.8	0.8
Income of old-aged	0.2	0.2
Natural rate of interest	0.68	0.720541
Government debt	0.404852	0.372062

The effects of a 40% increase in the borrowing constraint of the young on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

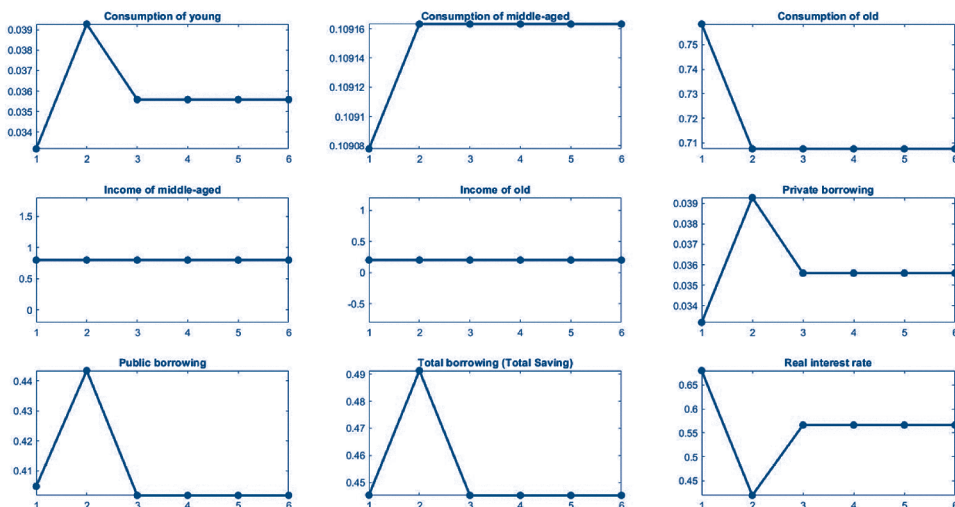
change, fiscal spending must be accompanied by higher public borrowing to maintain the government’s budget constraint. Higher borrowing translates into a higher loan demand, which should translate into a higher interest rate. However, as can be seen in Figure 4, consumption of the middle-aged and young goes up, while the consumption of the old goes down. This can only occur if the interest rate goes down, because it entices the young to borrow, and demotivates the middle-aged from saving for the future. Thus, for a certain period the interest rate overshoots its steady-state value. Nevertheless, in the second period, the natural rate of interest starts going up to clear asset markets and to maintain the equality of the budget constraint (since it has the lagged value of public debt and interest rate on the right-hand side), private borrowing decreases (which results in a reduction in the consumption of the young), public borrowing decreases and consequentially so does total borrowing.

Since public borrowing comes down to its initial level after having increased temporarily, government spending results in an increase in private consumption based on private borrowing and on an increase in the consumption of the middle-aged

at the expense of the old. We can therefore conclude that government spending carries re-distributive effects on the consumption of different age cohorts and, moreover, due to its effect on the natural rate of interest, has effect on the private debt levels of young households (the increase in the consumption of the young is partially caused by the reduction in the consumption of the old). Looking at Table 4, we can examine that government spending does indeed have negative effects on the natural rates of interest as expected. This means that government spending needs to be accompanied by a permanent increase in government debt (as stated in section 3) to help counter the effects of population growth rate decline and/or increase in relative income inequality. In any other case, fiscal spending can exacerbate the effects of secular stagnation and result in an even lower natural rate of interest. Table 4 thus reiterates that *an increase in government spending, if unaccompanied by a permanent increase in public debt, results in a decrease of the natural rate of interest.*

Let us now turn to the two remaining instruments of fiscal policy, which is taxes levied on the middle-aged and taxes levied on the old. Taxes di-

Figure 4 » A 20% increase in fiscal spending



The effects of a 20% increase in fiscal spending on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

Table 4 » A 20% increase in fiscal spending

Variable	Initial steady state	Final steady state
Consumption of young	0.0331846	0.0334038
Consumption of middle-aged	0.109078	0.109086
Consumption of old	0.758333	0.75341
Borrowing of young	0.0331846	0.0334038
Savings of middle-aged	0.445338	0.44533
Income of middle-aged	0.8	0.8
Income of old-aged	0.2	0.2
Natural rate of interest	0.68	0.668975
Government debt	0.404852	0.404945

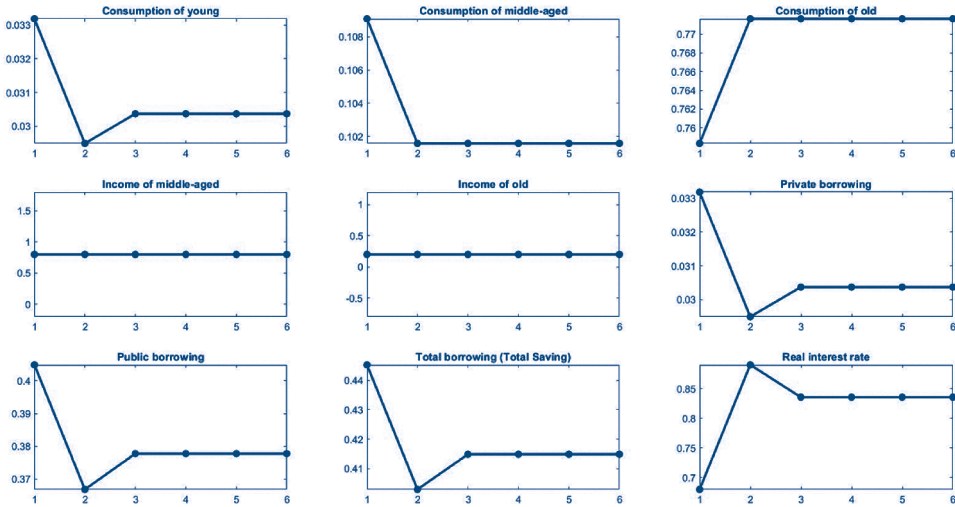
The effects of a 20% increase in fiscal spending on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

rected to the middle-aged cohort should intuitively lead to a reduction in the supply of loans (an increase of savings to pay the taxes) and thus to an increase in the natural rate of interest. Figure 5 below details the effects that an increase in taxes on

the middle-aged has on the model's endogenous variables. As Figure 5 shows, an increase in taxes on the middle-aged leads to a decrease in the consumption of the young and middle-aged, which is in line with the budget constraint of the middle-



→ **Figure 5** » *A 20% increase in taxes on the middle-aged*



The effects of a 20% increase in taxes on the middle-aged on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

**Table 5** » *A 20% increase in taxes on the middle-aged*

Variable	Initial steady state	Final steady state
Consumption of young	0.0331846	0.0303727
Consumption of middle-aged	0.109078	0.101589
Consumption of old	0.758333	0.771655
Borrowing of young	0.0331846	0.0303727
Savings of middle-aged	0.445338	0.41486
Income of middle-aged	0.8	0.8
Income of old-aged	0.2	0.2
Natural rate of interest	0.68	0.835534
Government debt	0.404852	0.377805

The effects of a 20% increase in taxes on the middle-aged on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

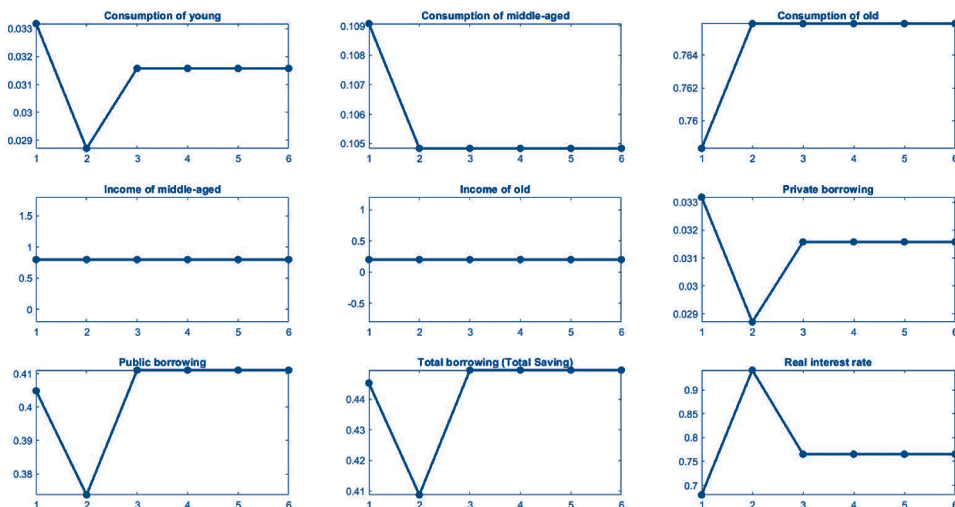
aged. Since consumption of the old increases and consumption of the middle-aged decreases (even though total savings go down), the interest rate has to overshoot its steady-state equilibrium value to maintain consumer optimization given by the Eul-

er equation (9), and to maintain equilibrium in the asset markets. Since savings of the middle-aged go down, but consumption of the old increases, it has to be the case that the interest payments accruing to the old must cover the reduction in savings.

The increase in the interest rate leads to a reduction in private and public borrowing and a reduction of the consumption of the young. The interest rate in the second period starts falling to its steady-state value in line with the government

budget constraint. This leads to a slight increase in public and private borrowing, and in the consumption of the young. The new interest rate is now higher than it initially was. Fiscal policy can, by taxing the middle-aged, increase the natural rate of

**Figure 6 » A 20% increase in taxes on the old-aged**



The effects of a 20% increase in taxes on the old on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

**Table 6 » A 20% increase in taxes on the old-aged**

Variable	Initial steady state	Final steady state
Consumption of young	0.0331846	0.0334038
Consumption of middle-aged	0.109078	0.109086
Consumption of old	0.758333	0.75341
Borrowing of young	0.0331846	0.0334038
Savings of middle-aged	0.445338	0.44533
Income of middle-aged	0.8	0.8
Income of old-aged	0.2	0.2
Natural rate of interest	0.68	0.668975
Government debt	0.404852	0.404945

The effects of a 20% increase in taxes on the old on consumption of the young, consumption of the middle-aged, consumption of the old, income of the middle-aged, income of the old, private borrowing, public borrowing, total borrowing, and the real rate of interest.

interest, albeit decreasing theirs and the young's consumption. The results in Table 5 below confirm our hypothesis, which is that ***an increase in the taxation of the middle-aged cohort leads to an increase in the natural rate of interest.***

Lastly, let us focus our attention on the effects of an increase in taxes, but now directed at the old-aged households.

We can intuitively assume that an increase in taxes will be accompanied by an increase in the natural rate of interest, since the middle-aged need to increase their supply of loans to save for the future so they can pay their taxes.

Intriguingly enough, the consumption of the old goes up in period 1, and the consumption of the middle-aged goes down. Since the interest rate adjusts to the decisions of households at each period, the Euler equation suggests that the interest rate must increase by more than its steady-state value. The overshooting of the interest rate causes private borrowing and public borrowing to decline. This leads to a decline in the consumption of the young. The interest rate in the second period starts to fall to its steady-state value in line with the increment of loan supply. This fall in the interest rate causes total borrowing to increase as evident from Equations (32) and (34), respectively. Public borrowing however in this peculiar case converges to a higher value than that at which it was before the tax was levied. The results in Table 6 reaffirm our theoretical model, which suggests that ***an increase in taxes on the old leads to an increase in the natural rate of interest.***

We can hence confirm that fiscal and macroprudential policy can help alleviate the natural rate of interest and help an economy out of secular stagnation. The relaxation of the borrowing constraint leads to an increase in the consumption of the young at the expense of the middle-aged and the old. Moreover, it decreases public borrowing,

which might make it politically more feasible to persuade policy makers. On the other hand, an increase in private borrowing might not be the best alternative, as it can lead to other economic imbalances in the future especially with respect to financial markets. Concerning fiscal policy, an increase of taxes on both the middle-aged and the old does indeed lead to an increase in the natural rate of interest. The middle-aged taxation might seem to be the most viable option, since it does not need to be met with such large increases in public debt as compared to taxing the old. Furthermore, taxing the middle-aged results in a larger increase in the natural rate of interest than when taxing the old. Concerning government spending, however, it needs to be financed either by taxes, a permanent increase in public debt, or a combination of both. Otherwise, an increase in government spending might exacerbate the negative effects of decreasing population growth and increasing income inequality.

## 6. Conclusion

Over the past forty years, natural rates of interest have been on a steady decline, productivity growth has been sluggish, and inflation has been well below central banks' 2% targets<sup>3</sup>, as is documented in the literature. It might come as a surprise, but dynamic stochastic general equilibrium models used for economic policy decision-making purposes do not account for such circumstances. In those models, the natural rate of interest cannot be permanently negative since it is determined by the discount factor of individuals. In an environment of low productivity, below target inflation and low or negative natural rates of interest – defined as secular stagnation, economic policy needs to shift its attention to models considering those realities.

We argue for an alternative. A theoretical mod-

<sup>3</sup> As of the writing of this article, inflation has overshoot the central banks' target. The authors however believe that this is not a consequence of long-term secular forces, but rather of a policy mismanagement of scarce natural resources and loose monetary policy conducted previously.

el of natural rates of interest based on an analysis of the demand and supply of loans. The framework of our analysis is based on an overlapping generations model that allows fundamental forces, such as population growth and income inequality, to have a significant role in determining the natural rates of interest. We present a quantitative analysis of an endowment economy, which confirms the negative effects that declining population growth and increasing income inequality have on natural rates of interest. Moreover, we utilize this framework to illustrate the capabilities of fiscal policy in tackling and aiding an economy out of secular stagnation using taxation and fiscal spending. Macroprudential policy, in the form of a relaxation of borrowing constraints of individuals, can also play a significant role in alleviating the burdens of negative secular forces.

The purpose of this article was to illustrate the long-term determinants of natural rates of interest on a simple endowment economy model. We understand that such a model cannot account for the precise realities observed, but that was not the in-

tention of the authors. The intention was to present to the reader a viable simplistic alternative to present-time more complicated models. The setting of the model presented allows for several extensions that might make it more feasible for monetary and fiscal policy conduct. For simplicity, we assumed that the borrowing constraint takes the form of a constant. However, the constraint can be updated to reflect asset prices and other real-world determinants of consumer's borrowing constraints. Moreover, including production and prices would certainly make the model more robust. Lastly, the model can be applied to the euro area and help explain investment-saving imbalances occurring between the northern members and southern ones with possible remedies.

We hope that future research pays more attention to the new challenges facing our economies and that it approaches alternatives as viable options, rather than proceeding in developing economic models that fail to reflect contemporary economic realities.

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## A Model of Secular Stagnation: A Quantitative Analysis for an Endowment Economy with Fiscal Policy

### ABSTRACT

*This paper follows the ideas developed and formulated by Eggertsson, Mehrotra and Robbins of a secular stagnation model in which persistently low or even negative natural rates of interest become a permanent state of the economy. This differs substantially from New Keynesian models which only see negative interest rates as a temporary occurrence. With monetary policy constrained by the zero lower bound, economic policy necessitates a further exploration of fiscal policy. We present a quantitative analysis of an endowment economy OLG model with fiscal policy to illustrate the capabilities of fiscal policy in aiding an economy out of secular stagnation.*

### KEYWORDS

*Secular stagnation; Interest rate; Fiscal policy; Overlapping generations model*

### JEL CLASSIFICATION

*E43; E52; E62; E32; C68*

